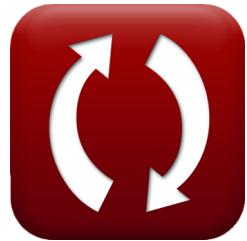




[calculatoratoz.com](http://calculatoratoz.com)



[unitsconverters.com](http://unitsconverters.com)

# Control System Design Formulas

Calculators!

Examples!

Conversions!

Bookmark [calculatoratoz.com](http://calculatoratoz.com), [unitsconverters.com](http://unitsconverters.com)

Widest Coverage of Calculators and Growing - **30,000+ Calculators!**

Calculate With a Different Unit for Each Variable - **In built Unit Conversion!**

Widest Collection of Measurements and Units - **250+ Measurements!**

Feel free to SHARE this document with your friends!

*[Please leave your feedback here...](#)*



© [calculatoratoz.com](http://calculatoratoz.com). A [softusvista inc.](#) venture!



# List of 31 Control System Design Formulas

## Control System Design ↗

### 1) Angle of Asymptotes ↗

**fx**

$$\phi_k = \frac{(2 \cdot (\text{modulus}(N - M) - 1) + 1) \cdot \pi}{\text{modulus}(N - M)}$$

[Open Calculator ↗](#)

**ex**

$$5.834386\text{rad} = \frac{(2 \cdot (\text{modulus}(13 - 6) - 1) + 1) \cdot \pi}{\text{modulus}(13 - 6)}$$

### 2) Bandwidth Frequency given Damping Ratio ↗

**fx**

$$f_b = \omega_n \cdot \left( \sqrt{1 - (2 \cdot \zeta^2)} + \sqrt{\zeta^4 - (4 \cdot \zeta^2) + 2} \right)$$

[Open Calculator ↗](#)

**ex**

$$54.96966\text{Hz} = 23\text{Hz} \cdot \left( \sqrt{1 - (2 \cdot (0.1)^2)} + \sqrt{(0.1)^4 - (4 \cdot (0.1)^2) + 2} \right)$$

### 3) Damped Natural Frequency ↗

**fx**

$$\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2}$$

[Open Calculator ↗](#)

**ex**

$$22.88471\text{Hz} = 23\text{Hz} \cdot \sqrt{1 - (0.1)^2}$$



## 4) Damping Ratio given Critical Damping ↗

$$fx \quad \zeta = \frac{C}{C_c}$$

[Open Calculator ↗](#)

$$ex \quad 0.100334 = \frac{0.6}{5.98}$$

## 5) Damping Ratio given Percentage Overshoot ↗

$$fx \quad \zeta = -\frac{\ln\left(\frac{\%_o}{100}\right)}{\sqrt{\pi^2 + \ln\left(\frac{\%_o}{100}\right)^2}}$$

[Open Calculator ↗](#)

$$ex \quad 0.100106 = -\frac{\ln\left(\frac{72.9}{100}\right)}{\sqrt{\pi^2 + \ln\left(\frac{72.9}{100}\right)^2}}$$

## 6) Damping Ratio or Damping Factor ↗

$$fx \quad \zeta = \frac{c}{2 \cdot \sqrt{m \cdot K_{\text{spring}}}}$$

[Open Calculator ↗](#)

$$ex \quad 0.188147 = \frac{16}{2 \cdot \sqrt{35.45 \text{kg} \cdot 51 \text{N/m}}}$$



**7) Delay Time** ↗

$$fx \quad t_d = \frac{1 + (0.7 \cdot \zeta)}{\omega_n}$$

**Open Calculator** ↗

$$ex \quad 0.046522s = \frac{1 + (0.7 \cdot 0.1)}{23\text{Hz}}$$

**8) First Peak Overshoot** ↗

$$fx \quad M_o = e^{-\frac{\pi \cdot \zeta}{\sqrt{1-\zeta^2}}}$$

**Open Calculator** ↗

$$ex \quad 0.729248 = e^{-\frac{\pi \cdot 0.1}{\sqrt{1-(0.1)^2}}}$$

**9) First Peak Undershoot** ↗

$$fx \quad M_u = e^{-\frac{2 \cdot \zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$$

**Open Calculator** ↗

$$ex \quad 0.531802 = e^{-\frac{2 \cdot 0.1 \cdot \pi}{\sqrt{1-(0.1)^2}}}$$

**10) Gain-Bandwidth Product** ↗

$$fx \quad G.B = \text{modulus}(A_M) \cdot \text{BW}$$

**Open Calculator** ↗

$$ex \quad 56.16\text{Hz} = \text{modulus}(0.78) \cdot 72\text{b/s}$$



**11) Number of Asymptotes** ↗

$$fx \quad N_a = N - M$$

[Open Calculator ↗](#)

$$ex \quad 7 = 13 - 6$$

**12) Number of Oscillations** ↗

$$fx \quad n = \frac{t_s \cdot \omega_d}{2 \cdot \pi}$$

[Open Calculator ↗](#)

$$ex \quad 6.365281\text{Hz} = \frac{1.748\text{s} \cdot 22.88\text{Hz}}{2 \cdot \pi}$$

**13) Peak Time** ↗

$$fx \quad t_p = \frac{\pi}{\omega_d}$$

[Open Calculator ↗](#)

$$ex \quad 0.137307\text{s} = \frac{\pi}{22.88\text{Hz}}$$

**14) Peak Time given Damping Ratio** ↗

$$fx \quad t_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

[Open Calculator ↗](#)

$$ex \quad 0.137279\text{s} = \frac{\pi}{23\text{Hz} \cdot \sqrt{1 - (0.1)^2}}$$



**15) Percentage Overshoot ↗**

$$fx \quad \%_o = 100 \cdot \left( e^{\frac{-\zeta \cdot \pi}{\sqrt{1-(\zeta^2)}}} \right)$$

**Open Calculator ↗**

$$ex \quad 72.92476 = 100 \cdot \left( e^{\frac{-0.1 \cdot \pi}{\sqrt{1-(0.1)^2}}} \right)$$

**16) Q-Factor ↗**

$$fx \quad Q = \frac{1}{2 \cdot \zeta}$$

**Open Calculator ↗**

$$ex \quad 5 = \frac{1}{2 \cdot 0.1}$$

**17) Resonant Frequency ↗**

$$fx \quad \omega_r = \omega_n \cdot \sqrt{1 - 2 \cdot \zeta^2}$$

**Open Calculator ↗**

$$ex \quad 22.76884 \text{Hz} = 23 \text{Hz} \cdot \sqrt{1 - 2 \cdot (0.1)^2}$$



**18) Resonant Peak** ↗

**fx**  $M_r = \frac{1}{2 \cdot \zeta \cdot \sqrt{1 - \zeta^2}}$

**Open Calculator** ↗

**ex**  $5.025189 = \frac{1}{2 \cdot 0.1 \cdot \sqrt{1 - (0.1)^2}}$

**19) Rise Time given Damped Natural Frequency** ↗

**fx**  $t_r = \frac{\pi - \Phi}{\omega_d}$

**Open Calculator** ↗

**ex**  $0.125507s = \frac{\pi - 0.27\text{rad}}{22.88\text{Hz}}$

**20) Rise Time given Damping Ratio** ↗

**fx**  $t_r = \frac{\pi - (\Phi \cdot \frac{\pi}{180})}{\omega_n \cdot \sqrt{1 - \zeta^2}}$

**Open Calculator** ↗

**ex**  $0.137073s = \frac{\pi - (0.27\text{rad} \cdot \frac{\pi}{180})}{23\text{Hz} \cdot \sqrt{1 - (0.1)^2}}$

**21) Rise Time given Delay Time** ↗

**fx**  $t_r = 1.5 \cdot t_d$

**Open Calculator** ↗

**ex**  $0.06s = 1.5 \cdot 0.04s$



**22) Setting Time when Tolerance is 2 Percent** **Open Calculator** 

$$fx \quad t_s = \frac{4}{\zeta \cdot \omega_d}$$

$$ex \quad 1.748252s = \frac{4}{0.1 \cdot 22.88Hz}$$

**23) Setting Time when Tolerance is 5 Percent** **Open Calculator** 

$$fx \quad t_s = \frac{3}{\zeta \cdot \omega_d}$$

$$ex \quad 1.311189s = \frac{3}{0.1 \cdot 22.88Hz}$$

**24) Steady State Error for Type 1 System** **Open Calculator** 

$$fx \quad e_{ss} = \frac{A}{K_v}$$

$$ex \quad 0.064516 = \frac{2}{31}$$

**25) Steady State Error for Type 2 System** **Open Calculator** 

$$fx \quad e_{ss} = \frac{A}{K_a}$$

$$ex \quad 0.060606 = \frac{2}{33}$$



## 26) Steady State Error for Type Zero System

**fx**  $e_{ss} = \frac{A}{1 + K_p}$

[Open Calculator !\[\]\(71ceb62b681518c82e95d615e7265d66\_img.jpg\)](#)

**ex**  $0.060606 = \frac{2}{1 + 32}$

## 27) Time of Peak Overshoot in Second Order System

**fx**  $T_{po} = \frac{(2 \cdot k - 1) \cdot \pi}{\omega_d}$

[Open Calculator !\[\]\(fc3a57079704ef1b99671c8cafae23be\_img.jpg\)](#)

**ex**  $1.235766s = \frac{(2 \cdot 5 - 1) \cdot \pi}{22.88\text{Hz}}$

## 28) Time Period of Oscillations

**fx**  $T = \frac{2 \cdot \pi}{\omega_d}$

[Open Calculator !\[\]\(d5831b2ac75eb48b4c49d27e61d24c03\_img.jpg\)](#)

**ex**  $0.274615s = \frac{2 \cdot \pi}{22.88\text{Hz}}$



## 29) Time Response in Overdamped Case

**fx****Open Calculator **

$$C_t = 1 - \left( \frac{e^{-\left(\zeta_{\text{over}} - \left(\sqrt{\left(\zeta_{\text{over}}^2\right) - 1}\right)\right) \cdot (\omega_n \cdot T)}}{2 \cdot \sqrt{\left(\zeta_{\text{over}}^2\right) - 1} \cdot \left(\zeta_{\text{over}} - \sqrt{\left(\zeta_{\text{over}}^2\right) - 1}\right)} \right)$$

**ex**  $0.807466 = 1 - \left( \frac{e^{-\left(1.12 - \left(\sqrt{\left((1.12)^2\right) - 1}\right)\right) \cdot (23\text{Hz} \cdot 0.15\text{s})}}{2 \cdot \sqrt{\left((1.12)^2\right) - 1} \cdot \left(1.12 - \sqrt{\left((1.12)^2\right) - 1}\right)} \right)$

## 30) Time Response in Undamped Case

**fx****Open Calculator **

**ex**  $1.952818 = 1 - \cos(23\text{Hz} \cdot 0.15\text{s})$

## 31) Time Response of Critically Damped System

**fx****Open Calculator **

**ex**  $0.858732 = 1 - e^{-\omega_n \cdot T} - \left(e^{-\omega_n \cdot T} \cdot \omega_n \cdot T\right)$



## Variables Used

- $\%_o$  Percentage Overshoot
- $A$  Coefficient Value
- $A_M$  Amplifier Gain in Mid Band
- $BW$  Amplifier Bandwidth (*Bit Per Second*)
- $c$  Damping Coefficient
- $C$  Actual Damping
- $C_c$  Critical Damping
- $C_t$  Time Response for Second Order System
- $e_{ss}$  Steady State Error
- $f_b$  Bandwidth Frequency (*Hertz*)
- $G.B$  Gain-Bandwidth Product (*Hertz*)
- $k$  Kth Value
- $K_a$  Acceleration Error Constant
- $K_p$  Position of Error Constant
- $K_{spring}$  Spring Constant (*Newton per Meter*)
- $K_v$  Velocity Error Constant
- $m$  Mass (*Kilogram*)
- $M$  Number of Zeroes
- $M_o$  Peak Overshoot
- $M_r$  Resonant Peak
- $M_u$  Peak Undershoot
- $n$  Number of Oscillations (*Hertz*)



- $N$  Number of Poles
- $N_a$  Number of Asymptotes
- $Q$  Q Factor
- $T$  Time Period for Oscillations (*Second*)
- $t_d$  Delay Time (*Second*)
- $t_p$  Peak Time (*Second*)
- $T_{po}$  Time of Peak Overshoot (*Second*)
- $t_r$  Rise Time (*Second*)
- $t_s$  Setting Time (*Second*)
- $\zeta$  Damping Ratio
- $\zeta_{over}$  Overdamping Ratio
- $\Phi$  Phase Shift (*Radian*)
- $\Phi_k$  Angle of Asymptotes (*Radian*)
- $\omega_d$  Damped Natural Frequency (*Hertz*)
- $\omega_n$  Natural Frequency of Oscillation (*Hertz*)
- $\omega_r$  Resonant Frequency (*Hertz*)



# Constants, Functions, Measurements used

- **Constant:** **pi**, 3.14159265358979323846264338327950288  
*Archimedes' constant*
- **Constant:** **e**, 2.71828182845904523536028747135266249  
*Napier's constant*
- **Function:** **cos**, cos(Angle)  
*Trigonometric cosine function*
- **Function:** **ln**, ln(Number)  
*Natural logarithm function (base e)*
- **Function:** **modulus**, modulus  
*Modulus of number*
- **Function:** **sqrt**, sqrt(Number)  
*Square root function*
- **Measurement:** **Weight** in Kilogram (kg)  
*Weight Unit Conversion* 
- **Measurement:** **Time** in Second (s)  
*Time Unit Conversion* 
- **Measurement:** **Angle** in Radian (rad)  
*Angle Unit Conversion* 
- **Measurement:** **Frequency** in Hertz (Hz)  
*Frequency Unit Conversion* 
- **Measurement:** **Bandwidth** in Bit Per Second (b/s)  
*Bandwidth Unit Conversion* 
- **Measurement:** **Stiffness Constant** in Newton per Meter (N/m)  
*Stiffness Constant Unit Conversion* 



## Check other formula lists

- **Control System Design Formulas** 
- **Modelling of Electrical Systems Formulas** 
- **Transient and Steady State Response Formulas** 

Feel free to SHARE this document with your friends!

### PDF Available in

[English](#) [Spanish](#) [French](#) [German](#) [Russian](#) [Italian](#) [Portuguese](#) [Polish](#) [Dutch](#)

10/3/2023 | 6:27:10 AM UTC

[Please leave your feedback here...](#)

